A NOTE ON RAO, HARTLEY AND COCHRAN'S METHOD

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INTRODUCTION :

Rao, Hartley and Cochran [3] proposed a method of selecting a sample of size n using random groups (RHC Scheme) and suitable estimator $\hat{Y}_{RHC}$. If $N=nR+k$, k groups are of $R+1$ and $n-k$ groups are of size R then

$$V(\hat{Y}_{RHC}) = \frac{1}{n}\left[1-\frac{n-1}{N-1}+\frac{k(n-k)}{N(N-1)}\right]\sigma^2 \qquad \ldots(1)$$

where $\sigma^2 = \sum_{i=1}^{N} Y_i^2/p_i - Y^2$ [Notations have usual meaning]

If n units are drawn by probability proportional to size with replacement (PPSWR) method and estimator of Y is

$$Y_{PPSWR} = \frac{1}{n}\left(\sum_{i=1}^{n} \frac{Y_i}{p_i}\right)$$

then $V(\hat{Y}_{PPSPR}) = \sigma^2/n$ $\qquad \ldots(2)$

Comparing (1) and (2) it is often concluded that RHC scheme is better than PPSWR scheme. However, this comparison is not proper since expected effective sample sizes in these two cases are different. Singh and Kishore [4] considered PPSWR method with different number of units and compared. Again their comparison is not completely meaningful as they ignored the integer requirement. In this paper compasrion had been done using the technique of Ramkrishanan [2] and treating number of units in PPSWR as integer valued random variable such that expected effective sample size is n;

Main Result :

If m units are drawn by PPSWR method then expected effective sample size is given by

$$f(m)=N-\sum_{i}(1-p_i)^m$$

It is rare to find integer m such that $f(m)=n$. We also note that for every integer m, $f(m) \leqslant f(m+1)$ Let r be the integer such that

$$f(r)< n \leqslant f(r+1)$$

and let $p=[f(r+1)-n]/[f(r+1)-f(r)]$.

The modified PPSWR method (Mod PPSWR) is as follows :

(*i*) With probability p choose a sample by drawing r units by PSSWR method, or

(*ii*) with probability $(1-p)$ choose a sample by drawing $(r+1)$ units by PPSWR method

For this Mod PPSWR merhod expected effective sample size is given by

$$p\,f(r)+(1-p)\,f(r+1)$$

which on simplification reduces to n.

For this method we propose the following unbiased estimator.

$\hat{Y}_1=\{\sum_{i=1}^{d} y_i/p_i\}/d$ where d=number of drawn units and

$$V(\hat{Y}_1)=\frac{r+p}{r(r+1)}\sigma^2 \qquad \ldots(3)$$

Comparison :

From (1) and (3) we see that RHC scheme is better than Mod PPSWR scheme if for given $(p_1, p_2 \ldots p_N)$

$$\frac{1}{n}\left[1-\frac{n-1}{N-1}+\frac{k(n-k)}{N(N-1)}\right] \leqslant \frac{r+p}{r(r+1)} \qquad \ldots(4)$$

It is very difficult to conclude that this condition will hold good universally. Therefore, the following example illustrates the point.

Data, are about number of inhabitants in (000s) in 49 cities in year 1920 [taken From Cochran (1)] and relevant computations are as follows.

n	k	*LHS oF* (4)	r	p	*RHS oF* (4)
2	1	0.48979	2	0.95559	0.49260
3	1	0.31971	3	0.86429	0.32202
4	1	0.23469	4	0.72366	0 23618

Note that RHC scheme is better than Mod PPSWR scheme.

REFERENCES

[1] *Cochran W.G.* (1963) : Sampling Technique (Second Ed.) *John Wiley and Sons New York.*

[2] *Ramkrishnan M.K,* (1969) : Some results on the comparison of simple random sampling with and without replacement Sankhya (A) 31, 333-342.

[3] *Rao J.N.K., Hartley H.O.* and *Cochran W.G.* (1962) : On a simple procedure of unequal probability sampling without replacement JRSS (B) 24, 482-491.

[4] *Singh R.* and *Kishore L.* (1975): On Rao Hartley and Cochran's method of sampling Sankhya (C), 37, 88-94.